

Fig. 1 Plot of the (radius)<sup>2</sup> vs time from the experiment of Ref. 1.

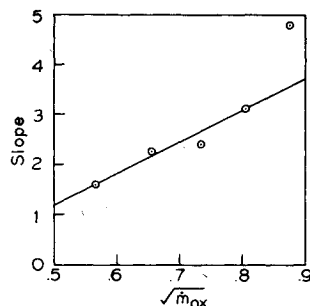


Fig. 2 Plot of the slope with  $\sqrt{\dot{m}_{ox}}$  from data of Ref. 2.

account of  $O/F$  variation. These methods of refining the curve fit are not discussed here.

Figure 9 (Ref. 1), which shows the plot of  $\log a$  vs  $\log n$  with time, indicates that  $a$  and  $n$  vary widely with time. This cannot be correct if the convective relation is meaningful. It is very likely that the data processing technique leading to  $a$  and  $n$  is incorrect.

Reference 2 treats the same problem in the case of aluminized propellants. This reference seems to use fundamental relations which are different from those in Ref. 1 and which are again incorrect. The heat balance relation is written as

$$\dot{Q}_w = \dot{Q}_T - (\dot{Q}_{re} + \dot{Q}_p)$$

where  $\dot{Q}_T$  is taken as total heat produced at the flame zone and  $\dot{Q}_p$  is the heat taken away by the particulate combustion products. In the event  $\dot{Q}_{re} + \dot{Q}_p = 0$ , which happens to occur at  $t=0$  in the formulation<sup>2</sup> (a fact which should again be pointed out as being unclear), one obtains  $\dot{Q}_w = \dot{Q}_T$ .

The total heat produced at the flame zone written in terms of fluxes is  $\rho_f \dot{r} H$  where  $H$  is the heat of combustion. Thus we obtain for the case of zero particle content

$$\dot{Q}_w = \rho_f \dot{r} h_{eff} = \rho_f \dot{r} H$$

a relation which leads to the absurdity that  $h_{eff} = H$ .

The implication is that term  $\dot{Q}_T$  should not be the total heat released at the flame zone, but the heat transferred to the surface by convection and radiation itself. Also, as pointed out earlier,  $\dot{Q}_{re}$  is irrelevant to the heat balance relation. Again the correct heat balance relation to be used is the one given by Eq. (1).

A few other features of this paper which are different from Ref. 1 though equally incorrect are as follows. The linearity of  $(\dot{r}_0 - \dot{r}_t)$  vs  $1/r$  plot and increased slope with oxidizer flow rate are taken as support for the predominance of radiation. In fact the authors conclude that with increased mass flow rate contribution to radiant heat flux increases. The reason given is that the flame temperature increases with mass flow rate. First, the flame temperature has no relation to the mass flow rate. In laminar boundary layer (which appears to be more relevant in the present case) the flame gets located at the position where stoichiometry of fuel and oxidizer is established and the flame temperature is adiabatic flame temperature at all flow rates.<sup>4</sup> The increase in oxidizer flow

rate only affects the flame position and not the flame temperature. Second, the statement that radiant flux increases with mass flow rate is unacceptable in the view of evidence available in other studies.<sup>3</sup>

Third, the linearity of the plot of  $(\dot{r}_0 - \dot{r}_t)$  vs  $1/r$  and the increased slope with mass flow rate is simply explained using the convective theory as will be demonstrated.

We have  $\dot{r}_t = aG^n$ , from which we obtain  $\dot{r}_0 - \dot{r}_t = aG^n - aG^{0n}$  where  $G^0$  is the mass flux at  $t=0$ . Further if we take  $G^0 = 4\dot{m}_{ox}/\pi d^2$ , a fact which is valid for large  $O/F$ , we obtain

$$\dot{r}_0 - \dot{r}_t = a \left[ \frac{4\dot{m}_{ox}}{\pi d_0^2} \right]^n - a \left[ \frac{4\dot{m}_{ox}}{\pi d^2} \right]^n = a \left[ \frac{4\dot{m}_{ox}}{\pi} \right]^n \left[ \frac{1}{d_0^{2n}} - \frac{1}{d^{2n}} \right]$$

If  $n=0.5$  for the laminar flow conditions one obtains a linear fit for  $(\dot{r}_0 - \dot{r}_t)$  vs  $1/r$  data. The slope of the curve fit is proportional to  $\dot{m}_{ox}$ . Thus, the increase in slope can be explained as due to the increase in  $\dot{m}_{ox}$ . To further substantiate this, the slopes of the curves in Fig. 8 of Ref. 2 were plotted against  $\dot{m}_{ox}$ . This plot appears in Fig. 2. Though the linearity of four of the five data points may be taken as support for the convective theory the present author hesitates to draw this conclusion in view of the possible incorrectness in the analysis leading to regression rates.

Summarizing, the essential conclusions drawn by the authors,<sup>1,2</sup> that radiant flux is a very important phenomenon in hybrid combustion, is a consequence of a combination of incorrect mathematics and implausible physics. The role of radiant flux in hybrid combustion has already been well elucidated by Wooldridge and Muzzy.<sup>3</sup>

#### Acknowledgment

The author takes this opportunity to thank the Aeronautical R and D Board for funding the project under which he is working and for according permission to publish this paper. The author is thankful to the members of Saturday Seminar for the invigorating discussions on the subject.

#### References

- <sup>1</sup>Rastogi, R.P., Kishore, K., and Chaturvedi, B.K., "Heterogeneous Combustion Studies of Polystyrene and Styrene Copolymer," *AIAA Journal*, Vol. 12, Sept. 1974, pp. 1187-1192.
- <sup>2</sup>Rastogi, R.P., Kishore, K., and Deepak, D., "Heterogeneous Combustion of Aluminized Propellants," *AIAA Journal*, Vol. 14, Feb. 1976, pp. 229-234.
- <sup>3</sup>Wooldridge, C.E. and Muzzy, R.J., "Internal Ballistic Considerations in Hybrid Rocket Design," *Journal of Spacecraft and Rockets*, Vol. 4, Feb. 1967, pp. 255-262.
- <sup>4</sup>Weinberg, F.J., "The First Half-Million Years of Combustion Research and Today's Burning Problems," *Fifteenth Symposium (International) on Combustion*, The Combustion Institute, Pittsburgh, Pa., 1974, p. 4.

#### Reply by Authors to P. J. Paul

R. P. Rastogi\* and Desh Deepak†  
University of Gorakhpur, Gorakhpur, India

THE comments of Paul on our recent papers on hybrid combustion<sup>1,2</sup> seem to originate from hasty conclusions due to lack of understanding of the basic theme of these contributions which has been emphasized too often in the relevant papers. These papers<sup>1,2</sup> stress the importance of radiative heat transfer to the combustion products and

Received Jan. 3, 1976.

Index categories: Fuels and Propellants, Properties of; Combustion in Heterogeneous Media; Solid and Hybrid Rocket Engines.

\*Senior Professor of Chemistry, Department of Chemistry, Associate Fellow AIAA.

†Senior Research Fellow, Department of Chemistry.

unreacted oxidizer,  $\dot{Q}_{re}$ , and *not* the radiative heat transfer to the fuel surface,  $\dot{Q}_r$ . Unfortunately, Paul has not been able to distinguish between these two types of radiative transfers, which are illustrated in Figs. 1 and 2. Hence, the comparison of results with Wooldridge and Muzzy<sup>3</sup> is irrelevant.

Equation (14) of Ref. 1 has been misquoted by Paul and should read as follows

$$\dot{r}\rho_f\Delta H = \dot{Q}_c - \dot{Q}_{re} \quad (1)$$

where  $\dot{Q}_c$  is the total heat which is being transferred from the flame to: 1) combustion products and unreacted oxidizer as well as to 2) the fuel surface. Thus,  $(\dot{Q}_c - \dot{Q}_{re})$  is simply the heat which is exclusively transferred to the fuel surface by convection and radiation, but the latter is generally assumed to be negligible especially in unmetallized propellants.<sup>4</sup> Thus, the observations of Paul on Eq. (14) are wholly unjustified.

It should be noted that Eq. (5) of Ref. 2

$$\dot{Q}_w = \dot{Q}_T - [\dot{Q}_{re} + \dot{Q}_p] \quad (2)$$

is the modified form of Eq. (1) where the additional term  $\dot{Q}_p$  is related to the heat taken away by the particulate combustion products and  $\dot{Q}_T$  and  $\dot{Q}_c$  have the same meaning. Owing to lack of appreciation of these points, Paul has created a chain of confusion. Equation (16) of Ref. 1 has been misquoted and should read as follows

$$\dot{Q}_{re} = \sigma\epsilon_w\epsilon_g T_r^4 \quad (3)$$

and not

$$\sigma\epsilon_w\epsilon_r T_r^4.$$

where  $\epsilon_w$  is the emissivity of the flame surface and *not* the fuel surface and hence comparison with Wooldridge and Muzzy<sup>3</sup> is unwarranted.

We now come to Eq. (23) of Ref. 1, which has again been misquoted and which should read as

$$\log [(\rho_f\Delta H/\sigma\epsilon_w p^{*\beta} T_r^4)(\dot{r}_t - \dot{r}_0) + 1] = -\alpha pz \quad (4)$$

and not

$$\log [\rho_f\Delta H/\sigma\epsilon_w p^{*\beta} T_r^4(\dot{r}_t - \dot{r}_c) + 1] = \alpha pz$$

There is nothing wrong with Eq. (4) since  $\dot{r}_t = \dot{r}_0$  when there is no chance of heat loss,  $\dot{Q}_{re}$ , due to radiation, i.e., when  $z=0$ . The only point is that this situation cannot be achieved experimentally. Hence, even if appropriate extrapolations are made  $(\dot{r}_0)_{\text{extrapolated}}$  would always be less than  $(\dot{r}_0)_{\text{true}}$ . Hence,

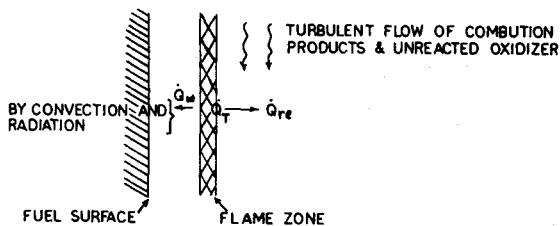


Fig. 1 Turbulent boundary-layer combustion of unmetallized fuel.

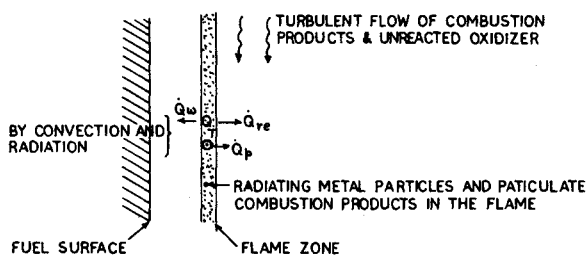


Fig. 2 Turbulent boundary-layer combustion of metallized fuel.

when the left-hand side is plotted against  $z$ , in Fig. 11 of Ref. 1, it may not pass through the origin as we find the case. The same remarks apply to Fig. 12 of Ref. 1.

Remarks about Eq. (26) of Ref. 1 are also unwarranted since the equation fits the experimental data in the time range studied. It has to be noted that no experimental data could be obtained for time  $t=0$ .

The alternative Eq. (2) suggested by Paul in his comments does not improve the matter and is unacceptable. The equation cannot be obtained by the integration of  $\dot{r}=aG^n$  noting that the total gas flow rate,  $\dot{m}_g$ , varies with time at a particular position in a cylindrical grain.<sup>3,4</sup> Similarly Fig. 1 of Paul also does not improve matters because it will also lead to an infinite value of  $\dot{r}$  at time  $t=0$  when  $r \rightarrow 0$  so that  $\dot{Q}_{re} \rightarrow 0$ .

Paul's remarks about Fig. 9 of Ref. 1 also originate on account of a similar misunderstanding. The relation  $\dot{r}=aG^n$  holds between  $\dot{r}$  and  $G$  for a particular time and  $a$  and  $n$  can change. Experiments were performed by varying the oxidizer mass flow rate  $\dot{m}_{ox}$  for a particular time.

Some self-contradictory statements have been made in the comment by Paul regarding the total heat produced in the flame zone while referring to Ref. 2 which lead to absurd conclusions since  $\dot{Q}_w$  represents the net heat transported to the fuel surface which, in the case of metallized fuels, is given<sup>3,5</sup> by

$$\dot{Q}_w = \rho_f(1-K)\dot{r}h_{v\text{eff}} \quad (5)$$

where  $\rho_f$  is the density of fuel,  $K$  the mass fraction of non-volatile surface material, and  $h_{v\text{eff}}$  the effective heat of gasification of solid phase.

Whereas in case of unmetallized fuels<sup>3,5</sup>

$$\dot{Q}_w = \rho_v\dot{r}h_{v\text{eff}} \quad (6)$$

where  $\rho_v$  is the bulk density of volatile component of fuel, and  $h_{v\text{eff}} = h_{vb}$ , where  $h_{vb}$  is the heat of gasification of binder material. This point has been clearly emphasized in Eqs. (1) and (4) of Ref. 2 but unfortunately this escaped the attention of Paul.

Doubt has been cast on our hypothesis as suggested in Ref. 2 that the flame temperature increases with oxidizer mass flow rate. This is substantiated by our experimental observation according to which flame temperature increases with  $\dot{m}_{ox}$  in the range studied. This is expected in view of the fact that greater extent of combustion would be achieved by higher  $\dot{m}_{ox}$ . Since flame temperature is found to increase with  $\dot{m}_{ox}$ , obviously the radiative flux would increase with  $\dot{m}_{ox}$  and hence the doubts do not stand.

The alternate equation suggested by Paul is unsound and untenable. It is obvious that

$$\dot{r}_0 - \dot{r}_t \neq aG^n - aG^{on}$$

Further,

$$G \neq 4\dot{m}_{ox}/\pi d^2, \text{ but } \approx 4\dot{m}_g/\pi d^2$$

Hence the suggested equation is incorrect. If the necessary corrections are made, his equation should read

$$\dot{r}_0 - \dot{r}_t = a \left[ \frac{4\dot{m}_g}{\pi} \right]^n \left[ \frac{1}{d_0^{2n}} - \frac{1}{d^{2n}} \right]$$

which is also not acceptable for the reason that  $\dot{m}_g$  depends both on position as well as time<sup>3,4</sup> and hence the equation cannot be used for any useful purpose and does not satisfy our plots (Figs. 8 and 10 of Ref. 2), which show an intercept.

## References

- Rastogi, R. P., Kishore, K., and Chaturvedi, B. K., "Heterogeneous Combustion Studies of Polystyrene and Styrene Copolymer," *AIAA Journal*, Vol. 12, Sept. 1974, pp. 1187-1192.

<sup>2</sup>Rastogi, R. P., Kishore, K., and Deepak, D., "Heterogeneous Combustion of Aluminized Propellants," *AIAA Journal*, Vol. 14, Feb. 1976, pp. 229-234.

<sup>3</sup>Wooldridge, C. E. and Muzzy, R. J., "Internal Ballistics Considerations in Hybrid Rocket Design," *Journal of Spacecraft and Rockets*, Vol. 4, Feb. 1967, pp. 255-262.

<sup>4</sup>Marxman, G. A., Wooldridge, C. E., and Muzzy, R. J., "Fundamentals of Hybrid Boundary Layer Combustion," *Progress in Astronautics and Aeronautics, Heterogeneous Combustion* edited by H. G. Wolfhard, I. Glassman, and L. Green Jr., Vol. 15, 1964, Academic Press, New York, pp. 485-522.

<sup>5</sup>Seifert, H. S., "Hybrid Rocket Theory and Design," *Jet, Rocket, Nuclear, Ion and Electric Propulsion: Theory and Design*, Vol. 7, ed. by W.H.T. Loh, Springer-Verlag, New York, 1968, pp. 332-335.

## Comment on "Resolution of Runge-Kutta-Nystrom Condition Equations through Eighth Order"

Richard H. Battin\*  
Charles Stark Draper Laboratory, Inc.  
Cambridge, Mass.

AT the time of publication of the subject paper,<sup>1</sup> the author was obliged to resort to a numerical experiment to eliminate two of the condition equations in deriving the general eighth-order, eight-stage algorithms. Since then the author has succeeded in developing an analytic argument which is, of course, far more satisfying and is now presented in this Comment.

As in the paper, we impose the constraints

$$b_1 = 0, \quad p_7 = 1$$

Then we form a set of five equations by adding Eqs. ( $\alpha^i$ ) and ( $\alpha^{i+2}$ ) and subtracting two times Eq. ( $\alpha^{i+1}$ ) for  $i=1, \dots, 5$ . When these are compared to Eqs. ( $\gamma^0$ ), ( $\epsilon^0$ ), ( $\iota^0$ ), ( $\sigma^0$ ), ( $e_2$ ), and we recall that  $H_j^0 = 0$ , it follows that

$$H_j^0 = \frac{1}{2} p_j (1 - p_j)^2 b_j \quad (j=2, \dots, 6)$$

Equation ( $e_5$ ) can now be written as

$$(1 - p_2)^2 A_2^4 b_2 + \dots + (1 - p_6)^2 A_6^4 b_6 = 2/8! \quad (e_5)$$

which is identical to the equation formed by adding Eqs. ( $\iota^0$ ) and ( $\iota^2$ ) and subtracting two times Eq. ( $\iota^1$ ). Therefore, Eq. ( $e_5$ ) may be discarded if the three equations (i) are satisfied by the Nystrom parameters. The same operations applied to Eqs. ( $\lambda^0$ ), ( $\lambda^1$ ), and ( $\lambda^2$ ) show that Eq. ( $e_6$ ) may also be discarded.

All minimal-stage Runge-Kutta-Nystrom algorithms through eighth order are now complete with the condition equations fully resolved analytically.

Finally, we note a typographical error in the paper on page 1016. Two sets of parameters  $p_0, p_1, p_2, p_3$  were cited which Nystrom used to develop fifth-order, four-stage algorithms. The first set should have been

$$p_0 = 0, \quad p_1 = \frac{2}{5}, \quad p_2 = \frac{2}{3}, \quad p_3 = \frac{4}{5}$$

Received Nov. 1, 1976.

Index category: Computer Technology and Computer Simulation Techniques.

\*Associate Department Head. Also Lecturer, Department of Aeronautics and Astronautics, Massachusetts Institute of Technology. Fellow AIAA.

The value for  $p_3$  was omitted in the typesetting.

## Reference

<sup>1</sup>Battin, R. H., "Resolution of Runge-Kutta-Nystrom Condition Equations through Eighth Order," *AIAA Journal*, Vol. 14, Aug. 1976, pp. 1012-1021.

## Comment on "Stress Concentration in the Plastic Range"

Ralph Papirno\*  
Army Materials and Mechanics Research Center,  
Watertown, Mass.

THE authors' rationale for using a semigraphical method for determining the plastic stress concentration factor,  $k_p$ , from the Neuber equation used with the Ramberg-Osgood stress-strain law, is the claim that the solution of the resulting equation requires a lengthy trial and error procedure.<sup>1</sup> One purpose of this Comment is to show that the procedure need not be "lengthy" when the equation is expressed in non-dimensional form. The time required for hand computation with a desk or pocket electronic calculator is directly related to the desired precision of the solution value. A simple root-finder routine has been used in our laboratory for machine computation as well. Another purpose of this Comment is to identify a limitation in calculating  $K_p$  which is a consequence of the limitation on the applicability of the Ramberg-Osgood equation and to refer to some earlier experimental results which were compared with the predictions of the Neuber theory.

Using the same notation as Ref. 1, the Neuber equation can be written as

$$K_{el}^2 = \sigma \epsilon / \epsilon_p \quad (1)$$

It is assumed that the reference stress,  $p$ , is a known quantity. By suitable algebraic manipulation, the Ramberg-Osgood approximation of the stress strain curve can be transformed to

$$\sigma \epsilon = (\sigma_y^2 / E) [ (\sigma / \sigma_y)^2 + (3/7) (\sigma / \sigma_y)^{m+1} ] \quad (2)$$

and similarly

$$p \epsilon_p = (\sigma_y^2 / E) [ (p / \sigma_y)^2 + (3/7) (p / \sigma_y)^{m+1} ] \quad (3)$$

Substitution of Eqs. (2) and (3) into Eq. (1) results in an equation of the form

$$Z^2 + (3/7) Z^{m+1} - B = 0 \quad (4)$$

Where  $Z = (\sigma / \sigma_y)$  and

$$B = K_{el}^2 [ (p / \sigma_y)^2 + (3/7) (p / \sigma_y)^{m+1} ] \quad (5)$$

The second term in the brackets in Eq. (5) is vanishingly small when  $p$  is elastic.

Numerical solutions, accurate to one part in a thousand, can be found in times comparable to that required for the procedure of Ref. 1 on a pocket electronic calculator. For machine computation, I have used a simple root-finder routine as part of a more extensive program for computation

Received Jan. 7, 1977.

Index category: Structural Static Analysis.

\*Research Mechanical Engineer, Mechanics Research Laboratory. Member AIAA.